Financial Literacy

How will we know if we have been successful in the teaching of Mathematics Literacy?

In large part the test must surely lie in whether or not the graduates are more effective: self managing individuals; contributing workers, and critical citizens.

While Mathematical Literacy is much broader than financial literacy – one hope of the Mathematical Literacy programme would be to have individuals making wiser financial decisions. Not everybody does: as we follow the news we read and hear about:

- The wealthy and the poor alike getting caught out by pyramid schemes.
- Workers going on strike demanding higher wages while the economy is at its weakest in many years and all around them people are losing their jobs.
- Cars, homes and other possessions being repossessed because people cannot make their loan repayments.
- People playing the Lotto in the hope of instant wealth.
- People who have not saved enough for their old age.

Millionaires hit by fraud scheme

Investors from as far abroad as the United States, Britain, Israel, Canada and the United Arab Emirates have lost millions of dollars which they invested in Barry Tannenbaum's pyramid scheme.

Twenty-two millionaires from the Western Cape are among the 400-odd South African investors who lost money in the collapsed scheme.

… The 22 Western Cape millionaires are estimated to have invested about R64.5m in the scheme, which promised enormous dividends.

www.fin24.com (17 June 2009)

Would these things happen if people were more Mathematically Literate?

The challenge, as I see it, is not so much to teach formulae and/or methods but to help students explore the parameters at play in the different scenarios and in so doing to develop an understanding of the impact of these parameters on the situations.

Cell phones and spreadsheets can play a very powerful role in such investigations. In the activities that follow we will be doing just that – using cell phones and spreadsheets to explore a number of situations.

Many financial situations, in particular loans and investments, can be explored using a simple "balance sheet" with the structure below.

% interest (p/a):	*rate at which interest is calculated – expressed as an annual percentage of the balance*

Period (date)	Interest	Instalment	Balance
	-	-	*opening balance*
Period 1	*calculated using previous balance and interest rate*	*payment/investment added to the account*	*previous balance + interest + installment*
Period 2			
Period 3			
Period 4			

- Interest rate: the cost of borrowed money expressed as a percentage
  - Note if interest is calculated quarterly or monthly then the rate is simply the annual interest rate divide by the number of times that interest is calculated in the year.
- Opening balance: the amount at the start:
  - In the case of a loan this will be the loan amount mostly expressed using a negative sign to indicate money owing.
  - In the case of an investment this will typically be zero or it may be some positive value – an initial saving.
- Interest: at the end/start of each period; interest is calculated based on the balance in the account at the end of the previous period:
  - In the case of a loan this interest represents the fee paid by the borrower to the lender (most often the bank) for the use of the borrowed money.
  - In the case of an investment this interest still represents the fee paid by the "borrower" – this time the bank – to the "lender" (investor) for the use of the borrowed money
- Instalment: an amount added to the account during each period
  - In the case of a loan this amount is intended to reduce the loan
  - In the case of an investment this amount increases the value of the investment/saving

This "balance sheet" can easily be set up on a spreadsheet or by means of a table on a piece of paper. The advantage of using a spreadsheet is that it allows the user to easily and quickly explore the impact of changing the parameters in the problem and hence studying their effect.

Investigation 1: Buying a car – at least three options

There are at least three different payment options implied by the advertisement above:

- Paying cash
- Paying as per the deal described in the advertisement
    - Use the balance sheet to investigate how the "balloon value" is determined.
    - Show how the total cost has been determined.
    - Consider the different options for paying the final balloon value
- Paying a larger monthly instalment and avoiding a balloon payment
    - Use the balance sheet to investigate how large this monthly payment should be.
    - Determine the amount using either the loan factor table on the next page or the loan calculator on your cell phone.

Compare each of the options in terms of its advantages and disadvantages.

		Number of years												
		2	3	4	5	6	7	8	9	10	15	20	25	30
Interest rate (p/a)	5%	43,87	29,97	23,03	18,87	16,10	14,13	12,66	11,52	10,61	7,91	6,60	5,85	5,37
	5,50%	44,10	30,20	23,26	19,10	16,34	14,37	12,90	11,76	10,85	8,17	6,88	6,14	5,68
	6%	44,32	30,42	23,49	19,33	16,57	14,61	13,14	12,01	11,10	8,44	7,16	6,44	6,00
	6,50%	44,55	30,65	23,71	19,57	16,81	14,85	13,39	12,25	11,35	8,71	7,46	6,75	6,32
	7%	44,77	30,88	23,95	19,80	17,05	15,09	13,63	12,51	11,61	8,99	7,75	7,07	6,65
	7,50%	45,00	31,11	24,18	20,04	17,29	15,34	13,88	12,76	11,87	9,27	8,06	7,39	6,99
	8%	45,23	31,34	24,41	20,28	17,53	15,59	14,14	13,02	12,13	9,56	8,36	7,72	7,34
	8,50%	45,46	31,57	24,65	20,52	17,78	15,84	14,39	13,28	12,40	9,85	8,68	8,05	7,69
	9%	45,68	31,80	24,89	20,76	18,03	16,09	14,65	13,54	12,67	10,14	9,00	8,39	8,05
	9,50%	45,91	32,03	25,12	21,00	18,27	16,34	14,91	13,81	12,94	10,44	9,32	8,74	8,41
	10%	46,14	32,27	25,36	21,25	18,53	16,60	15,17	14,08	13,22	10,75	9,65	9,09	8,78
	10,50%	46,38	32,50	25,60	21,49	18,78	16,86	15,44	14,35	13,49	11,05	9,98	9,44	9,15
	11%	46,61	32,74	25,85	21,74	19,03	17,12	15,71	14,63	13,78	11,37	10,32	9,80	9,52
	11,50%	46,84	32,98	26,09	21,99	19,29	17,39	15,98	14,90	14,06	11,68	10,66	10,16	9,90
	12%	47,07	33,21	26,33	22,24	19,55	17,65	16,25	15,18	14,35	12,00	11,01	10,53	10,29
	12,50%	47,31	33,45	26,58	22,50	19,81	17,92	16,53	15,47	14,64	12,33	11,36	10,90	10,67
	13%	47,54	33,69	26,83	22,75	20,07	18,19	16,81	15,75	14,93	12,65	11,72	11,28	11,06
	13,50%	47,78	33,94	27,08	23,01	20,34	18,46	17,09	16,04	15,23	12,98	12,07	11,66	11,45
	14%	48,01	34,18	27,33	23,27	20,61	18,74	17,37	16,33	15,53	13,32	12,44	12,04	11,85
	15%	48,25	34,42	27,58	23,53	20,87	19,02	17,66	16,63	15,83	13,66	12,80	12,42	12,25
	15,00%	48,49	34,67	27,83	23,79	21,15	19,30	17,95	16,92	16,13	14,00	13,17	12,81	12,64
	16%	48,72	34,91	28,08	24,05	21,42	19,58	18,24	17,22	16,44	14,34	13,54	13,20	13,05

Monthly repayment per R1 000,00 borrowed

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Investigation 2: Changes in home loan interest rate and period

At times when the economy goes through tough times people struggle to meet the monthly instalment obligations on their home loans. Rather repossessing the home, the bank will sometimes explore increasing the loan repayment period and thereby decreasing the monthly instalment.

- Use the balance sheet to investigate:
    - The impact of changes in the interest rate on monthly payments and total loan repayment value.
    - The impact of changes in the loan period on monthly payments and total loan repayment value.

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Investigation 3: Saving for retirement

Although retirement may seem a distant notion for some, for others it is a more immediate reality. However, as this investigation will reveal, you cannot start preparing for your retirement too early!

Consider the following scenario:

- You are 25 years old.
- You plan to work till you are 65 and then to retire.
- Once you retire you will need money for your daily expenses.
- You hope/expect to live to be at least 75 years old.

What fraction of your monthly salary should you put into savings as you plan for your retirement?

- Use the balance sheet to investigate the impact of the following on your retirement plan:
    - The difference between the average inflation rate over the period and the interest rate that you can secure for your savings.
    - The age at which you start saving for retirement and the age at which you want to retire.
    - The percentage (fraction) of your monthly net salary that you save.

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% interest (p/a):	

Period (date)	Interest	Instalment	Balance

% interest (p/a):	

Period (date)	Interest	Instalment	Balance

The impact of drinking on your ability to drive a car

The *Arrive Alive* campaign (www.arrivealive.co.za) is among other things concerned about communicating to people the dangers associated with drinking and driving. When somebody drives under the influence of alcohol they not only endanger their own lives but also those of other innocent people who may become involved in an accident caused by the inebriated driver.

In this activity we explore some of the "numbers" associated with drinking alcohol and blood alcohol levels – a measure used to determine how fit a person is to drive a car.

NOTE: There are a very large number of variables that are associated with drinking and blood alcohol levels; far too many for us to be able to incorporate as we explore this problem. That said; the underlying, if in-exact, principles apply and it is hoped that students will nonetheless gain a great deal from the experience of this investigation.

Background information

There are two legal ways of determining a person's blood alcohol level namely blood tests – which involve a sample of blood and breathalyzer tests – which involve a breath sample.

In the case of the blood test, 0,05g of alcohol per 100ml of blood is considered to be the legal limit. This means that if a driver's blood alcohol level exceeds this, the driver of the car is considered to be "over the limit". We summarise that as 0,05g/100ml.

There are two parts to our investigation. Firstly we need to be able to determine the alcohol content of an alcoholic beverage and secondly we need to have a way of determining how the alcohol consumed when drinking the beverage contributes to or determines a person's blood alcohol level.

Alcohol content of a drink

All alcoholic beverages have their alcohol content written on the container – this is usual expressed as some percentage per volume e.g. regular beer typically has an alcohol content of 5%per volume. This means that a 340ml can of beer will contain 5% × 340ml = 17ml of alcohol. The specific gravity of alcohol is in the order of 0,79g/ml and so a 340ml can with 5%/volume alcohol will contain 17ml × 0,79g/ml ≈ 13g of alcohol.

Blood alcohol level

A Swede, Erik Widmark developed a formula that can be used to estimate a person's blood alcohol level (BAL). The formula is known as the *Widmark Formula* (available at www.arrivealive.co.za) states that:

$$BAL = \frac{A}{p \times r \times 10}$$

Where: A is the amount of alcohol consumed in grams; p is the mass (weight) of the person; and r is the *Widmark* factor which is 0,7 for males and 0,6 for females.

The reason that males and females have a different *Widmark* factor is related to body fat. Body fat is a factor that impacts on alcohol absorption by the body and hence the blood, and in general women have a higher body fat content than men do.

Of course there are many other factors that how and how quickly the alcohol a person drinks converts into alcohol in your blood. These factors include:

- Whether or not the person has eaten before drinking;
- The kinds of food that the person has have eaten; and
- The percentage body fat of a person.

Recognising the limitations of the Widmark formula, we are now in a position to explore the relationship between the number of drinks consumed by a person and their blood alcohol level.

Activity

1. For each of the beverages listed in the table below, determine the number of grams of alcohol in a typical drink.

Beverage	Alcohol content	Typical volume of a single drink
Beer	5% per volume	340ml
Light beer	4,5% per volume	340ml
Wine	12% per volume	125ml
Spirits (vodka, brandy, whisky)	40% per volume	25ml

For the remainder of this activity we will regard a single drink – irrespective of what it is – to have an alcohol content of 10g.

2. Use the *Widmark* formula to determine the blood alcohol level of each of the following persons and hence determine whether or not they are over or under the limit:
   a. Male; 75 kg; 2 drinks
   b. Female; 60kg; 2 drinks

3. Use the *Widmark* formula to complete the following table.

	Weight	50 kg	60 kg	70 kg	80 kg	90kg	100kg	110kg	120kg
Number of drinks that the person can drink and still be under the limit	Male								
	Female								

4. Use the *Widmark* formula to complete the following table.

Number of drinks		1	2	3	4	5	6	7	8
Weight required for a person to be able to drink the number of drinks and still be under the limit	Male								
	Female								

Having drunk alcohol and achieved some blood alcohol level a person will again and after enough time reach a point where their blood alcohol level is zero. This is because the body eliminates alcohol over time. The accepted rate of elimination is considered to be approximately 0,015g/hour.

5. Use the *Widmark* formula and the elimination rate supplied to determine how long it will take each of the following people to regain the legal limit and to return to a state where their blood alcohol level is 0g/100ml.
   a. Male; 75 kg; 4 drinks
   b. Female; 60kg; 4 drinks

6. Discuss how we could use this context to:
   a. Teach the content of the Mathematical Literacy curriculum (i.e. link to Learning Outcomes and Assessment Standards);
   b. Teach about social responsibility – in particular: taking responsibility for ones actions; and
   c. Develop an assessment task for Mathematical Literacy (e.g. an investigation; an assignment or a project)

7. Discuss the impact of making choices and assumptions in mathematical modelling as demonstrated in this problem.

Building a Parabolic Reflector Dish

Parabolic dishes can increasingly be seen in and around homes in the suburbs of Cape Town in the form of receivers for satellite television. The function of the parabolic reflector dish is to focus the satellite signal received over some area onto a very small receiving antenna. The same principle is used in many other applications such as the design of automobile headlights (though in this application, the principle is reversed).

In this project you must build a parabolic reflector (diameter = 40 cm.) that will focus the Sun's rays onto a point. The temperatures that can be reached in this way using a well-built reflector will astound you and will be a powerful illustration of the role of the parabolic reflector dish in satellite television receivers.

To complete this task you must:

1. Read (or ask your teacher) about the focus—directrix definition for a parabola. This definition is quite important for you to be able to understand this task. The focus—directrix definition for the parabola leads to the following equation for a parabola, passing through the origin with axis of symmetry the y-axis, and focal point (0; c), which is in turn a useful equation for the cross section of the dish:

$$y = \frac{x^2}{4c}$$

   Your write-up must include some discussion of this definition and your understanding of it.

2. Choosing some value for c that will give you a pleasing shape for the parabola (try a few), you must build the parabolic reflector. You may want to get some ideas on building the reflector from the model that will be displayed in our classroom, but it is suggested that you adhere to the following steps:

   a) Using your equation and some graph paper, plot an accurate parabola graph of an appropriate size. Use this parabola to cut out the ribs (from a construction cardboard) onto which the dish will be built.

   b) Glue the ribs a maximum of 30° apart on some base and using further cardboard pieces design and build some strengthening struts between the ribs—your knowledge of the parabola's equation as well as the cosine rule will help you to design these supports quite accurately.

   c) Next design and cut some "wedges" of construction cardboard that will be glued to the ribs to form the reflector dish itself.

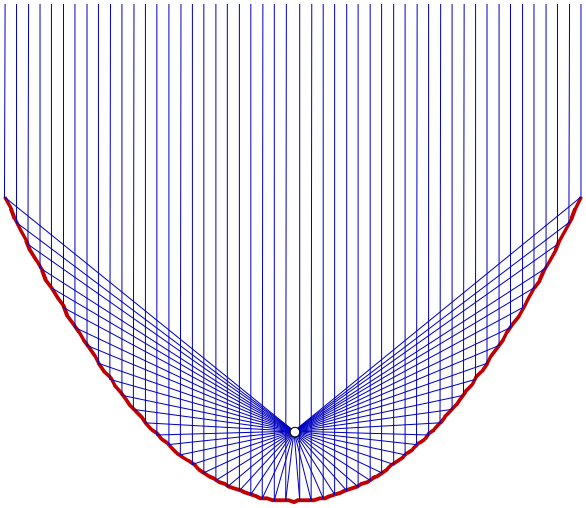
   d) Before gluing the wedges to the ribs cover them with aluminium foil taking care not to dirty the foil by touching it too much.

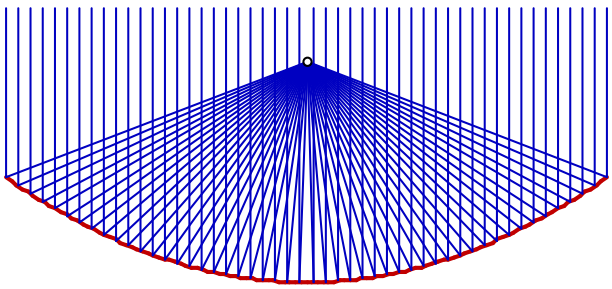
   e) You will be expected to hand in the dish, and your write up should describe how you determined the dimensions for the support struts and wedges.

3. Having built your reflector dish, you must test it. That is, you must determine what temperature(s) you can achieve at the focal point using the Sun as your energy source. NOTE: You should not be surprised if your reflector dish generates temperatures in excess of 100 °C and should hence use an appropriate thermometer. You may want to try some fun activity like frying a marshmallow!

Building a Parabolic Reflector Dish—Some Comments on the task

This project does not lend itself to a detailed solution—the instructions guide the student all the way. I will simply make some comments on the project based on the experience I had with it.





The Introduction

To really get the most out of this project it would be good for the teacher to give some input about the way in which a parabolic reflector focuses the incoming rays onto a focal point. It would be important for the teacher to discuss the focus-directrix definition for a parabola in this context and even to derive the formula given in the notes:. $y = \frac{x^2}{4c}$ I have a rather lovely demonstration of this on the computer that uses the program Geometer's Sketchpad that I am happy to make available.

The Task

Parts 2 a) and 2 b) are relatively easy and I have found that few if any students struggle with this section.

By contrast, part 2 c) is the challenge and the only really difficult part of the project. What most students do not realise is that while the angle between the support ribs may be 30° this is not the angle between the sides of the "wedges." This is of course the result of the wedges being on a curved surface and as we all know—angles around a point on a curved surface do not necessarily add up to 360°. Most students will only realise this when they have made ALL of their wedges, covered them in foil and stuck them down. What they then get is gaps in between the wedges. Students do not at first understand this and I have found that you need to guide them to think about this in a variety of ways. In my work I have not penalised students who had gaps in their model provided that they could explain why these had arisen and how they would design the wedge in the future to avoid this problem.

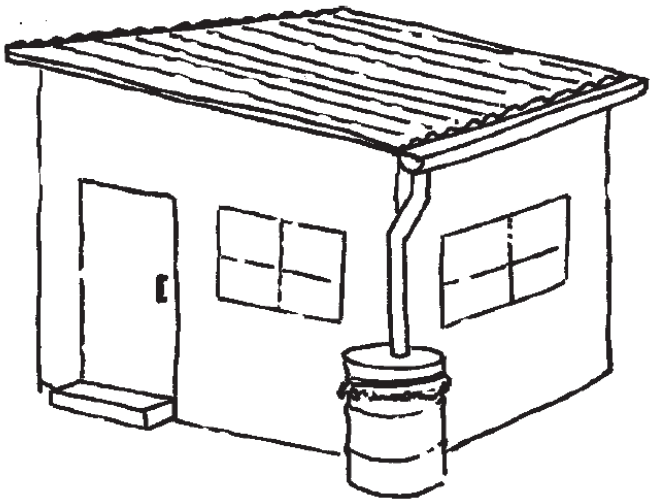
Part 3 of the project was generally very poorly done by my students. I had hoped that students would come to a realisation that the focal point is very much hotter than other points in the reflector even points very close to the focal point. I think that in future I would leave out this part and replace it with some

challenge to the student to demonstrate the power of their reflector—say setting a piece of newspaper alight.

A Possible Extension

Challenge students, using GSP or some other appropriate technology to develop a "programme" that prints out the different components of the model for different values of the parameter *c*. Students could even do this in lieu of building the reflector dish itself – although I suspect that they may want to assemble the dish to demonstrate the efficiency of their programme.

The realistic use of area and volume



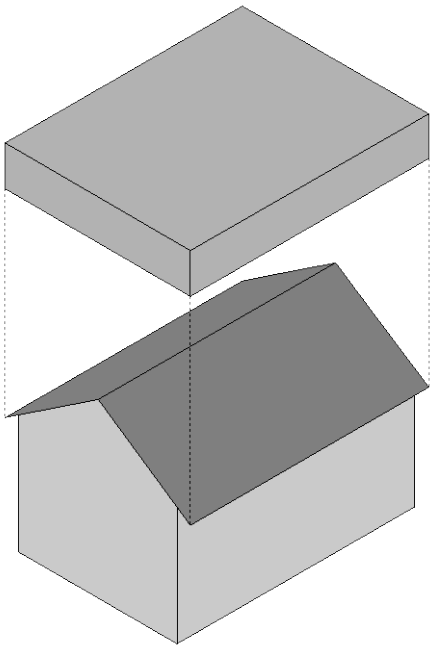
One of the challenges of developing lessons for the teaching of Mathematical Literacy is finding meaningful/realistic applications for some of the content in the Subject Statement. An example of a topic for which I have battled to find meaningful/realistic applications is area and volume. That is until I paged through a few textbooks and came across pictures of houses being used to collect water. Some research later and I had a rich context that involved a surprising amount of calculation involving area and volume – as well as relying on the ability to do conversions between different units etc.

Harvesting rainwater for household consumption

On useful resource that I found on the internet[1] highlighted the factors involved in this investigation. These include:

- Amount of rainfall (mm/year)
  The total amount of rainwater that can be collected in a year is determined by two things: (a) the average rainfall per month/year and (b) the area of the surface that will be used to collect the water.

  The South African Weather Service (http://www.weathersa.co.za) lists average rainfall figures for a number of towns from 1961 to 1990 on their website[2].



- Rainfall pattern

The nature of the rainfall pattern – seasonal rain vs. consistent rain throughout the year – impacts on the capacity of the storage container to be used.

- The area of the surface used to collect water (m^2)

In most cases the amount of water that can be collected is restricted/determined by the size of the roof.

The most interesting thing to understand here is that the shape (pointed vs. flat etc.) of the roof has no impact whatsoever on the amount of water that can be collected – we are only interested in the area of the "footprint" of the house. The sketch alongside makes this point.

[1] http://www2.warwick.ac.uk/fac/sci/eng/research/dtu/rwh/styles/
[2] http://www.weathersa.co.za/Menus/WXandClimate.jsp

- Storage capacity (m^3)
  A storage tank is needed to store the water and since this is likely to be the most expensive element in the project, it is important to plan the storage needs very carefully – the process is not unlike a budgeting process – water collected (income) vs. water consumed (expenses).

  What should be clear is that it is not necessary to store all of the water that can be collected in a year. Water will be used on a continual basis – the challenge is to get a sense of how large the container should be so that the surplus from months where collection exceeds consumption can be stored/saved for months where consumption exceeds collection (due to variations in rainfall patterns).

- Daily consumption rate (litres/person/day)
  It is important to have a sense of how much water each person in the household is likely to use each day as well as knowing how many people there are in the house.

  This could make an interesting data collection project – measuring the daily water consumption of a household. For a more general discussion it is useful to try and establish the typical water consumptions for individuals – although my research has revealed that these patterns vary enormously.
  - On source[3] suggests that people in South Africa who have to carry and fetch their own water use as little as 14liters/person/day
  - Another source[4] suggests that: "In 1995, in urban areas, people living in households connected to water sources used an average 43.4 cubic meters (m^3) per person per year, compared with urban dwellers without connections to water sources, who used 24.8 m^3 per person per year."

Water harvesting task

Use the information that follows to develop a task that can be used to teach applications of area and volume in a meaningful/realistic context.

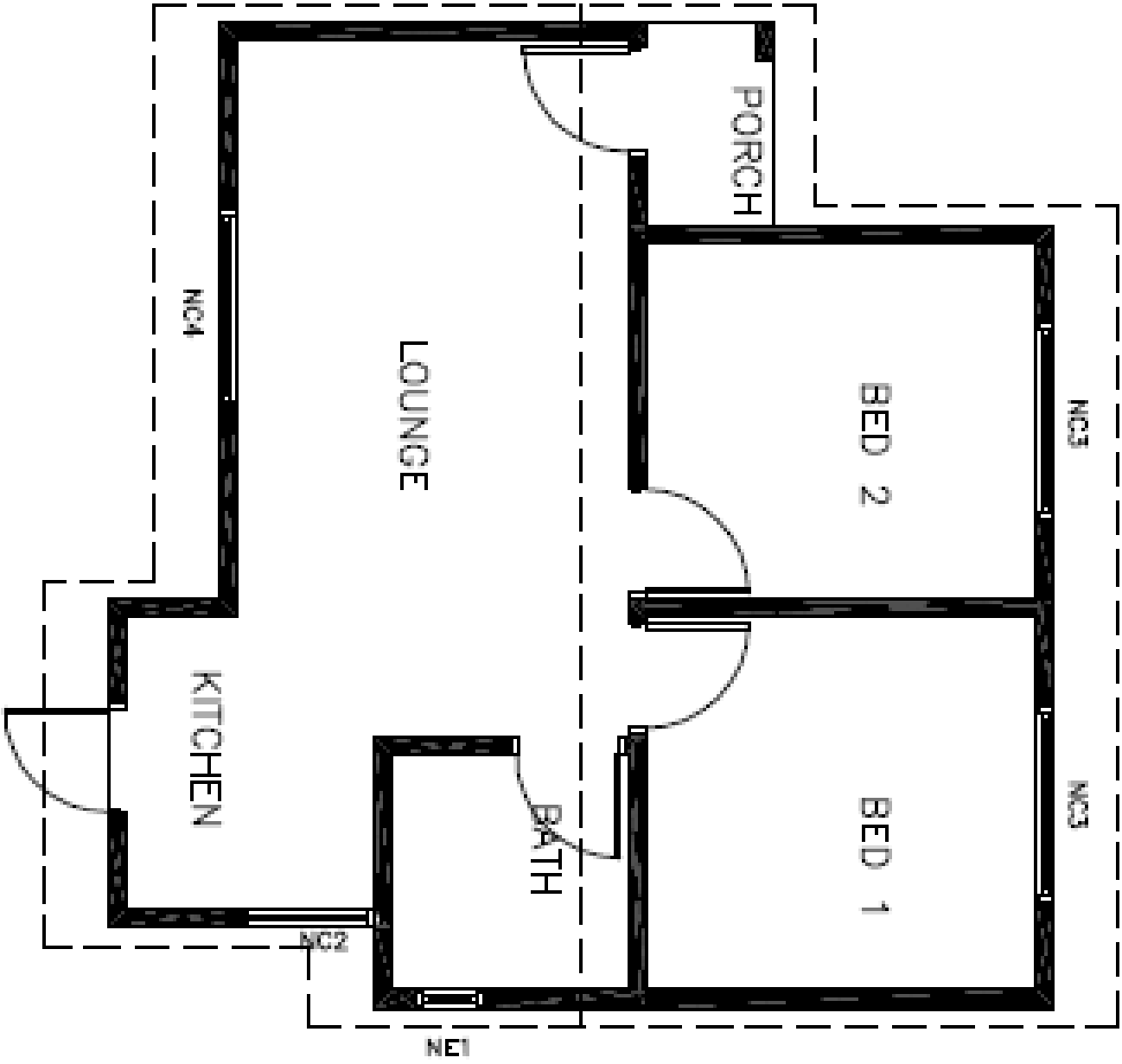
[3] academic.sun.ac.za/tsv/Centres/Egon/URDR%20annual%20report%202006.pdf
[4] http://www.ifpri.org/media/water_facts.htm

Average rainfall data 1961 to 1990 (http://www.weathersa.co.za/Menus/WXandClimate.jsp)

Province	Eastern Cape	Free State	Gauteng	Kza-Zulu Natal	Limpopo	Mpumalanga	North-West	Northern Cape	Western Cape	Western Cape
Town	Umtata	Bethlehem	Pretoria	Pietermaritzburg	Musina	Skukuza	Mmabatho	Calvinia	Beaufort West	Cape Town
January	87	96	136	141	58	94	117	14	35	15
February	89	77	75	117	57	96	83	12	30	17
March	83	94	82	113	39	66	74	26	30	20
April	58	58	51	48	27	38	57	27	20	41
May	18	9	13	24	10	14	14	22	11	69
June	11	12	7	13	4	11	5	34	8	93
July	18	7	3	11	1	11	3	23	9	82
August	15	27	6	31	1	8	5	24	14	77
September	35	35	22	60	12	28	13	13	12	40
October	73	83	71	74	24	40	37	11	21	30
November	75	96	98	104	49	63	64	13	27	14
December	88	86	110	108	57	92	67	9	19	17
Year	**650**	**680**	**674**	**844**	**339**	**561**	**539**	**228**	**236**	**515**

House plan

The scale of this plan is 1 : 50



Storage capacity needed

Month	Rainfall (mm)	Anticipated volume collected (l)	Anticipated consumption (l)	Surplus / deficit	Month-end Balance (l)
Jan					
Feb					
Mar					
Apr					
May					
Jun					
Jul					
Aug					
Sep					
Oct					
Nov					
Dec					
Jan					